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L4 – Future perspectives

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Fourier law

$$j_Q(x,t) = -k\frac{\partial T}{\partial x}$$

$$\frac{\partial j_Q}{\partial x} + \rho c\frac{\partial T}{\partial t} = 0$$

$$\boxed{\frac{\partial T}{\partial t} = D_T\frac{\partial^2 T}{\partial x^2}}$$

predicts an infinite speed (or velocity) of heat transfer (propagation)

Modified Fourier law

$$\tau\frac{\partial j_Q}{\partial t} + j_Q = -k\frac{\partial T}{\partial x},$$

τ is the thermal relaxation time

$$\tau\frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} = D_T\frac{\partial^2 T}{\partial x^2},$$

$$\boxed{\frac{1}{v_T^2}\frac{\partial^2 T}{\partial t^2} + \frac{1}{D_T}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2},}$$

gives the finite value of the heat transfer speed $\boxed{v_T = \sqrt{D/\tau}}$

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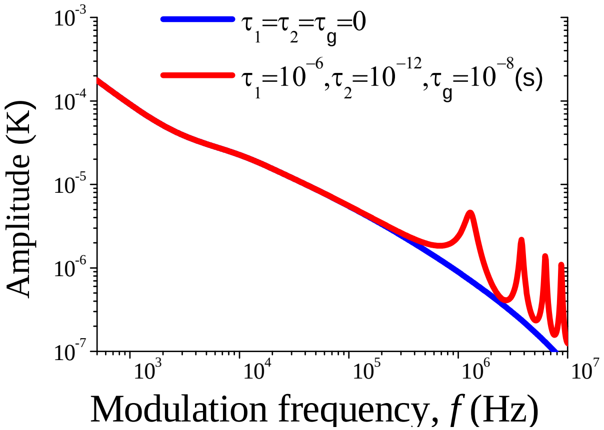
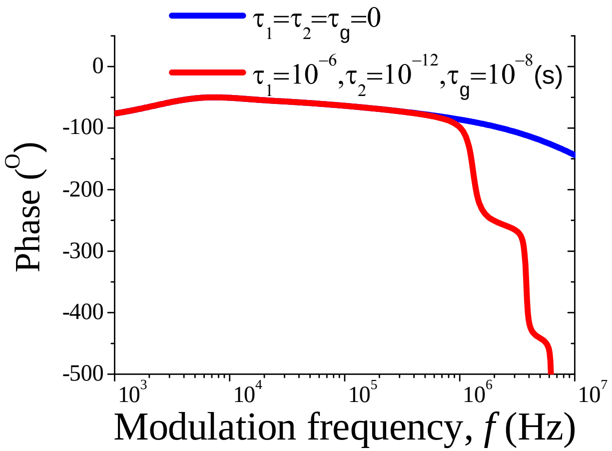
It can be observed that for low frequencies ($2\pi f\tau << 1$), the behavior of the spectra predicted by the Fourier and modified Fourier approach are similar, in contrast for higher frequencies ($2\pi f\tau >> 1$), where the modified Fourier effects are dominant.





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Fourier law

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The Fourier equation of heat conduction is fundamentally wrong because it assumes an infinite speed of propagation of heat, which is physically inadmissible.

It is a parabolic partial differential equation, i.e., a diffusion equation that includes a dissipative component, is always stable, and will always converge to steady state conditions after sufficient time.

Modified Fourier law

$$\tau \frac{\partial j_Q}{\partial t} + j_Q = -k \frac{\partial T}{\partial x},$$

$$\tau \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} = D_T \frac{\partial^2 T}{\partial x^2},$$

$$\boxed{\frac{1}{v_T^2}\frac{\partial^2 T}{\partial t^2} + \frac{1}{D_T}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2},}$$

$$\boxed{v_T = \sqrt{D/\tau}}$$

Based on the kinetic theory of gases i.e. microscopic aspects of lattice vibration, electrons transport, and their interactions, Fourier's linear heat flux, is modified to the given form, where τ is the thermal relaxation time of the heat conducting medium. Combining this modified flux and energy conservation law hyperbolic heat conduction equation (HHCE) is obtained.

Modified Fourier law

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This form is known as Maxwell's equation, because it resembles the equation of propagation of an electromagnetic field, i.e. light. However, the speed (velocity) of heat propagation, v_T, is not a fundamental property of the field, but is related to the mean free path of gas molecules. The idea is extended to solids, by assuming that heat is conducted by gas-like phonon or electron streams.

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HHCE describe production and propagation of thermal waves. Depending on boundary conditions, these waves may be overdamped or underdamped. Consequently, there will be cases where initial and boundary conditions may lead to thermal resonance, or temperature may overshoot to values higher than at the source.

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$$\boxed{v_T = \sqrt{D / \tau}}$$

(1) The speed of heat propagation v_T called by Landau the speed of second sound, is not the field property but merely a collection of various coefficients closely connected to the motion of phonons and electrons, obeying Newton's laws.

(2) Solving the HHCE it is possible to design boundary conditions such that heat would appear to be moving from a cold to a hot point, in violation of the second law of thermodynamics.

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Detail analysis shows that a HHCE-like equation can be derived directly from the theory of relativity, as a direct consequence of space–time duality (Minkowsky space), without any consideration of the microstructure of the heat-conducting medium (heat propagation speed becomes a field characteristics).

Modified Fourier law

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This approach results in an alternative expression for the heat flux vector that is more compatible with the second law of thermodynamics. Therefore, the RHCE brings the classical field theory of heat conduction into agreement with other branches of modern physics.

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The value of v_T can be much smaller in the case of non-homogeneous or thermally insulating material. In all cases, speed of heat is at least one order of magnitude less than speed of sound, which in itself is not very large in comparison with the heat source velocities applied by many manufacturing processes.

The present disadvantage is that there are no experimental methods for the accurate determination of v_T.

Modified Fourier law

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$$\boxed{v_T = \sqrt{D / \tau}}$$

Researchers have developed both numerical and analytical solutions to the modified Fourier equation through dimensional and parametric simplification. The solutions range from one-dimensional models with constant thermophysical properties to two-dimensional models with temperature-dependent thermophysical properties.

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Modified Fourier law

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Despite the improvement of this equation, it does not predict conduction at micro- or nanoscale levels. At these levels, the characteristic length of the system becomes smaller than the mean free path of the heat carrier. In conductors, the heat carriers are primarily electrons while the primary heat carriers in insulators and semi-conductors are quantized lattice vibrations knowns as phonons.

Despite their wave-like nature, phonons may be treated as particles when the structure size is larger than the wavelength of vibration. Under these conditions, the Boltzmann Transport Equation (BTE) models electron or phonon transport as

$$\boxed{\frac{\partial f}{\partial t^2} + \vec{v} \cdot \nabla f + \frac{\partial k_\omega}{\partial t} \cdot \nabla f_{k_\omega} = \left(\frac{\partial f}{\partial t} \right)_{collisions}}$$

where v is the carrier group velocity, k_ω is the wave vector, and f is the carrier distribution function which includes the carrier information (i.e. electrons vs. phonons)

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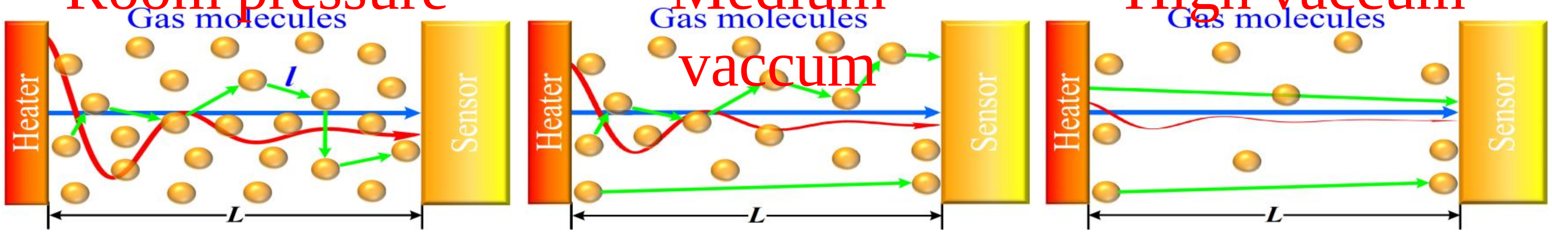
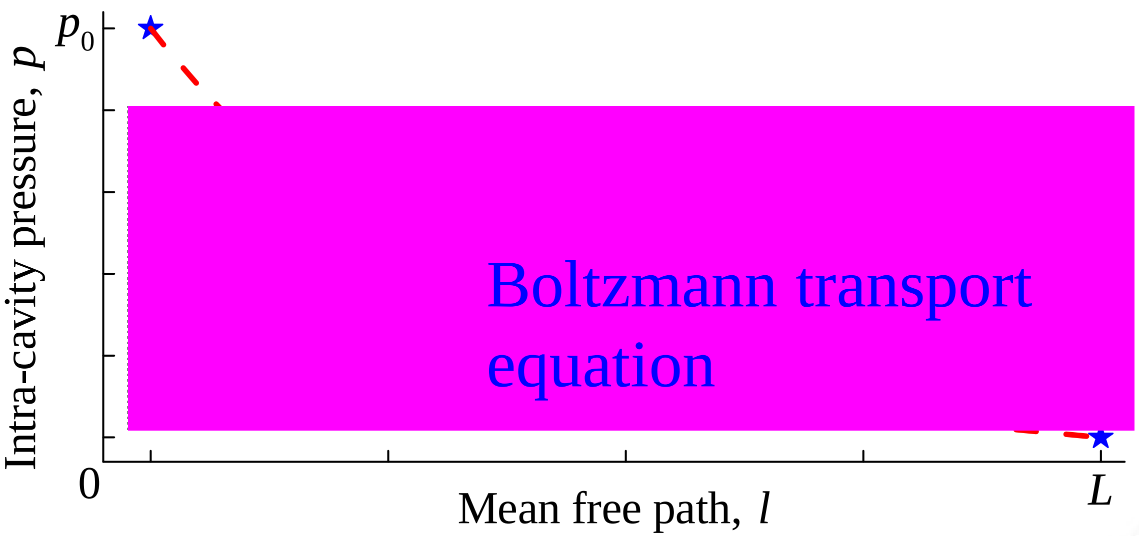
Room pressure | Medium vaccum | High vaccum



$l \quad L$	$l < L$	$l \approx L$
Diffusive regime	Diffusive+Ballistic regime	Ballistic regime



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The BTE is difficult to solve, thus two simplifications are commonly made: (1) the contribution of the wave vector is neglected and (2) the relaxation time approximation (RTA) is applied, yielding

$$\boxed{\frac{\partial f}{\partial t^2} + \vec{v} \cdot \nabla f = -\frac{f - f_0}{\tau(\omega)}}$$

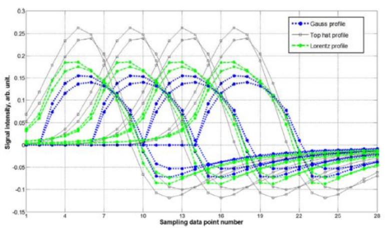
where f_0 is the equilibrium distribution, and the relaxation time, τ, is a function of the angular frequency, ω, of the heat carriers. The RTA assumes an effective relaxation time can be used when the number of carrier collisions is large, and its application is the typical starting point for transport analysis with the BTE.

Solving the BTE even in the case of simplest models in one-dimension requires lengthy numerical analysis. Some researchers has developed a set of ballistic-diffusive equations that only consider spatial and time variations. Other numerical methods have been developed such as the Lattice Boltzmann Method (LBM) that discretizes the BTE in the space, time and velocity domains.

The BTE, however, is limited by its treatment of electrons and phonons as classical particles. New theoretical models that account for the wave nature of electrons and phonons will be essential to advances in many fields that rely on lasers and nanostructures.

Experimental problems, data fitting & real-time operation

The fitting procedures are essential in experimental data analysis. Application of artificial intelligence could be a powerful tool not only to provide accurate and reliable results but to assure real-time operation, crucial for industrial application.



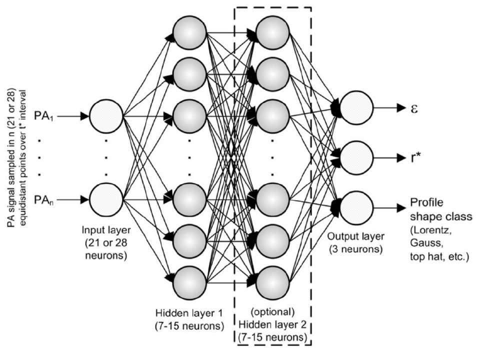
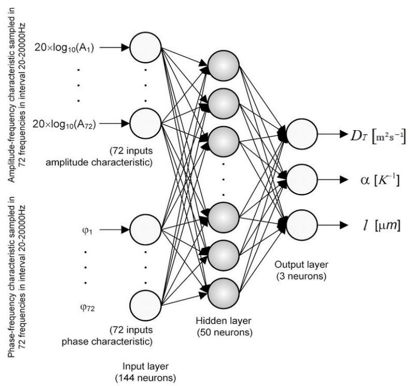
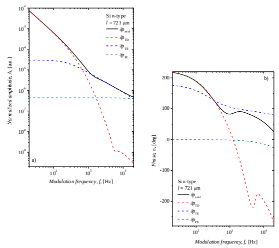
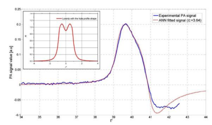


Figure 2. Neural network structure used for the real-time photoacoustic measurements.







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Experimental problems, data fitting & real-time operation

Applications of computational intelligence in photoacoustics are scarce; still some powerful results have been reported that demonstrate strong potential for use. For example, in an experiment of trace gas detection in a noisy environment, neural network application significantly improved pulsed photoacoustic detection by means of an adapted filter.

Experimental problems, data fitting & real-time operation

Novel approaches regarding the application of computational intelligence in photoacoustics are proposed. Feedforward multilayer perception networks, applied to simultaneous determination of laser beam profile and relaxation time, provide a high precision solution in real-time. Using this method, one can overcome the problem of possible unwanted variations in the laser beam spatial profile during the two consecutive pulses, which can significantly influence the relaxation time calculations.

Experimental problems, data fitting & real-time operation

Our experience in pulsed photoacoustics of gases tells us that ANNs are capable of recognizing the shape of the experimental signal and of simultaneously calculating different sample and set-up parameters reliably, accurately and in real time. Therefore, ANNs are applied within the frame of solid-state photoacoustics in the same manner as in the gases, acting as a powerful tool assigned for material characterization.

Experimental problems, data fitting & real-time operation

On the other hand, the proposed application of other AI tools like real-coded genetic algorithms (GAs) does not solve the problem in real-time, but it provides other attractive features.

Namely, GAs allow the efficient finding of the photoacoustic signal parameters that provide the best match with the experimental PA signal, with good precision and with parameters in a wide universe of discourse.

Experimental problems, data fitting & real-time operation

The promising obtained results in gases suggest that the application of fuzzy systems and the application of more complex hybrid neuro-fuzzy-genetic solutions might become important research directions in computationally intelligent photoacoustics.

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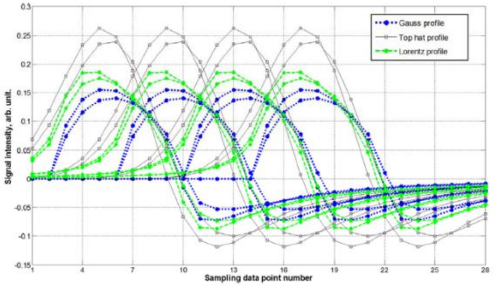

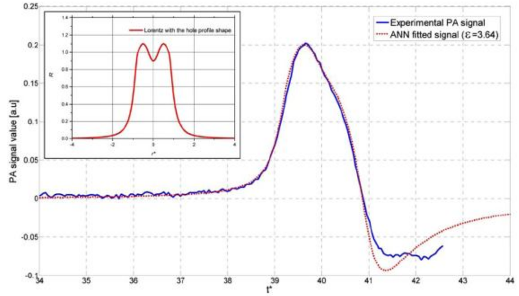

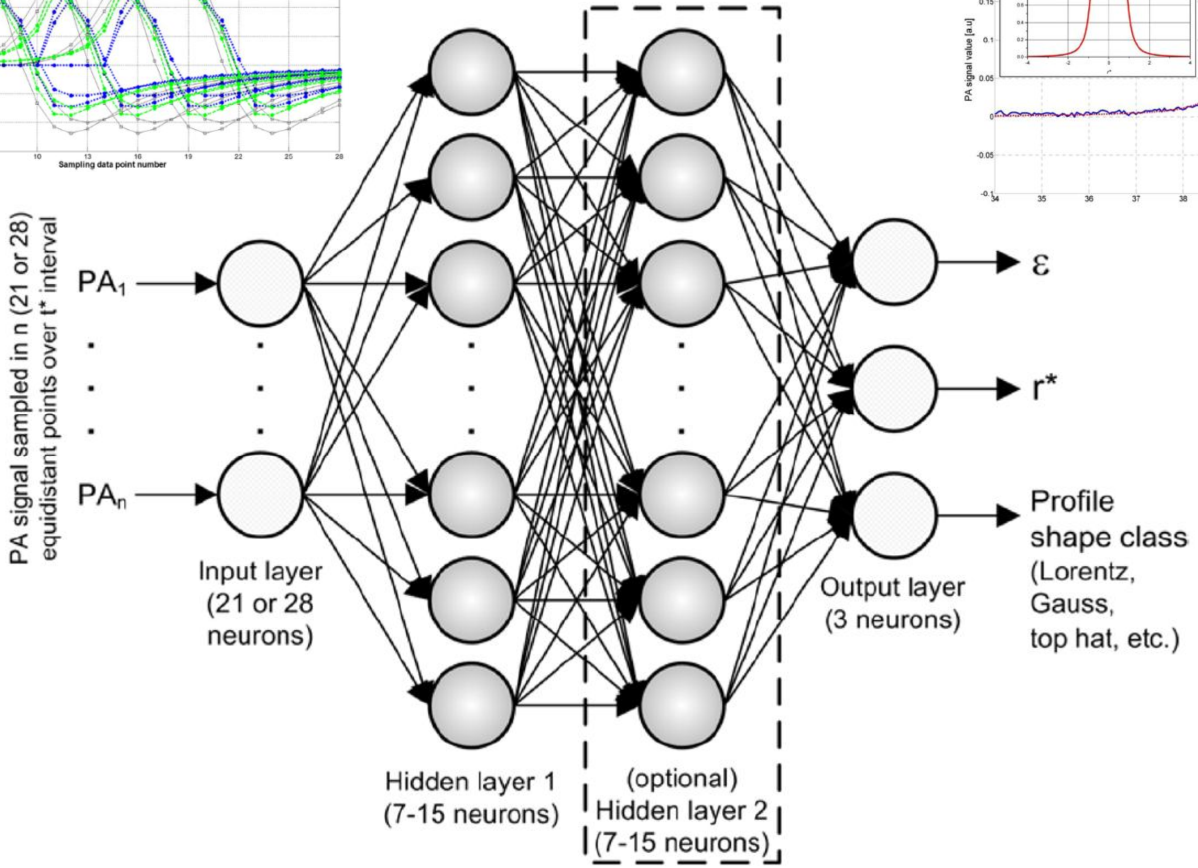

Figure 2. Neural network structure used for the real-time photoacoustic measurements.

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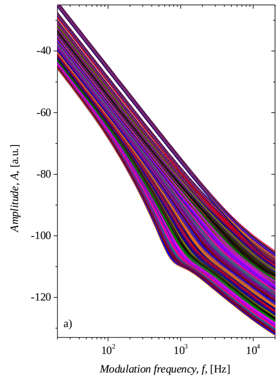

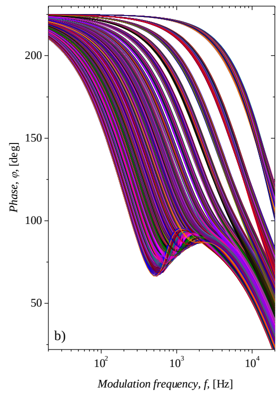

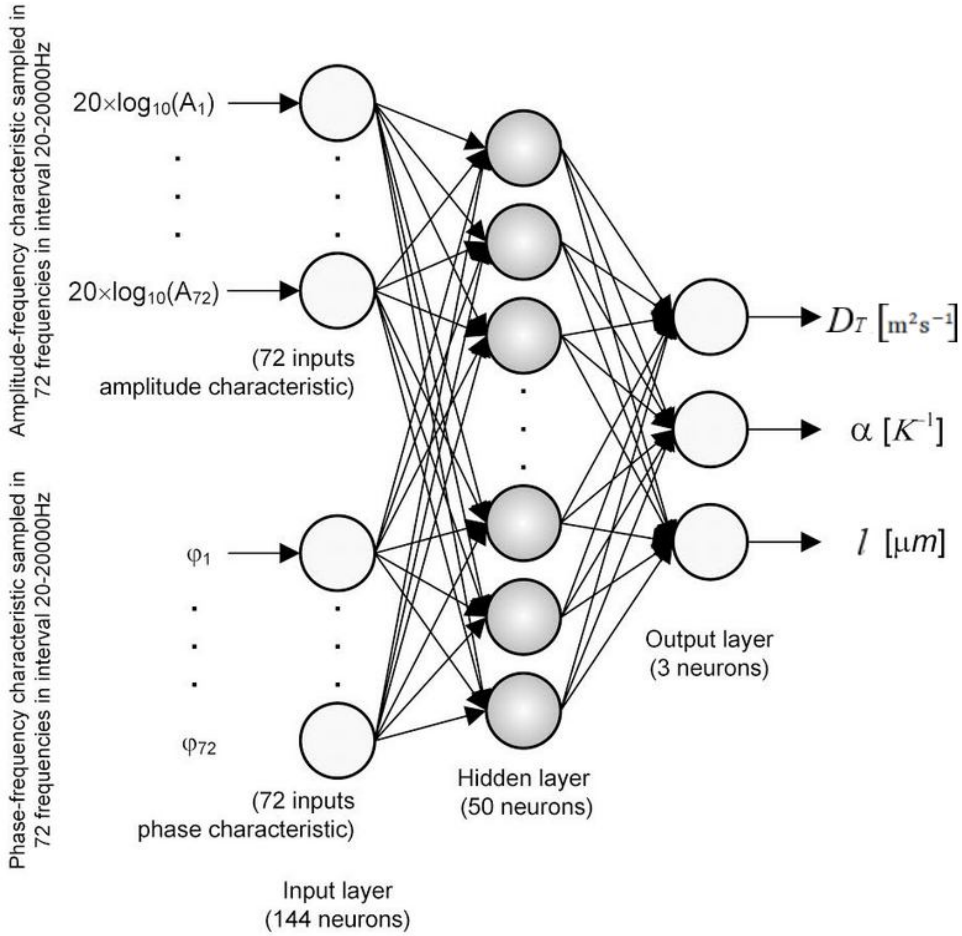

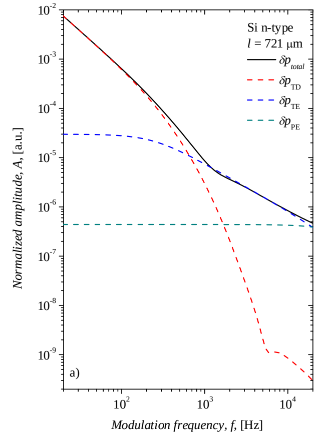

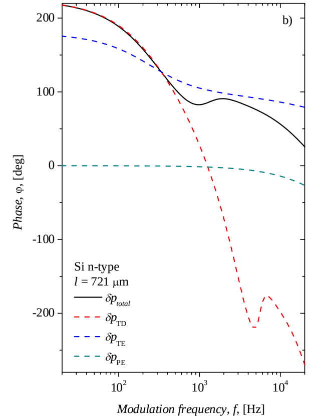

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